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Pythagorean Triangle With $\frac{\text{AREA}}{\text{PERIMETER}}$ As Quartic Integer

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***Abstract*— Four different patterns of Pythagorean triangle, where, in each of which the ratio may be expressed as a quartic integer. A few interesting relations among the solutions are given.**

***Index Terms*—Pythagorean triangle, Ratio (Area /perimeter) as quartic integer.**

I. INTRODUCTION

The method of obtaining three non-zero integers x, y and z under certain relations satisfying the equation $x^2 + y^2 = z^2$ has been a matter of interest to various mathematicians [1 to7]. In [8 to 13] special Pythagorean Problems are studied. In this communication, we present yet another interesting Pythagorean triangle where in each of which the ratio (Area/Perimeter) may be expressed as a quartic integer. A few interesting relation among the solutions are given.

***Notation**:*

$T_{m,n} = n\left[1+\frac{(n-1)(m-2)}{2}\right]$ = polygonal number of rank n, with sides m

Tet_n = Tetrahedral number of rank n = $\frac{n(n+1)(n+2)}{6}$.

SP_n = Square Pyramidal number of rank n = $\frac{n(n+1)(2n+1)}{6}$.

$P\rho_n$ = Oblong Number of rank n = $\frac{n^2(n+1)}{2}$

II. METHOD OF ANALYSIS

The most cited solution of the Pythagorean equation $x^2 + y^2 = z^2$ is

$x = 2pq$
$y = p^2 - q^2$ where $p > q > 0$ (1)
$z = p^2 + q^2$

Denoting the Area and the Perimeter of the above Pythagorean triangle by A and P respectively, the assumption that the ratio $\frac{A}{P}$ can be expressed as a quartic integer leads to the equation

$$pq - q^2 = 2\alpha^4 \qquad (2)$$

where α is the non-zero integer. ($\alpha > 0$)

Introducing the linear transformations

$$X = p + q$$
$$Y = p - q \qquad (3)$$

the equation (2) can be written as

$$Y(X - Y) = 4\alpha^4 \qquad (4)$$

In what follows, we present four different patterns of integral solutions of (4) and thus, in view of (3) and (1), the corresponding sides of the Pythagorean triangle are obtained

PATTERN I

Choosing

$$Y = 4\alpha \qquad (5)$$
$$X - Y = \alpha^3 \qquad (6)$$

in (4) and solving we get

$X = 4\alpha + \alpha^3$
$Y = 4\alpha$

In view of (3) the integral values p and q are given by

$$p = \frac{8\alpha + \alpha^3}{2},\ q = \frac{\alpha^3}{2}$$

where in α is even positive integer.

Case (i)

Taking $\alpha = 2k$; ($k > 0$)

we have $p = 8k + 4k^3$, $q = 4k^3$

and thus the corresponding sides of the Pythagorean triangle are obtained from (1) are given by

$x = x(k) = 64k^4 + 32k^6$
$y = y(k) = 64k^2 + 64k^4$
$z = z(k) = 64k^2 + 64k^4 + 32k^6$

Properties

1. $z - x$ is a perfect square.
2. $6(x - z)$ is a nasty number
3. $y - z + x \equiv 0 \pmod{64}$.

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4. $y - z + x$ = 64 times a quartic integer
5. $y - 128\ T_{3,k^2} = 0$
6. $y - 64k^2$ = 64 times a perfect square
7. $y - 64k^2 \equiv 0 \pmod{64}$
8. $z - 128\ T_{3,k^2} \equiv 0 \pmod{32}$
9. $z - 64k^2 - 32k^6$ = 64 times a perfect square
10. $y - z \equiv 0 \pmod{32}$
11. $x - z \equiv 0 \pmod{64}$
12. $x - y \equiv 0 \pmod{32}$
13. $z + 32k^4 = 192\,[\mathrm{Tet}_{k^2}]$.
14. $x + y - 32k^4 = 192\,[\mathrm{Tet}_{k^2}]$.
15. $y + z - 32k^4 - 64k^2 = 192\,[[\mathrm{Tet}_{k^2}]$.
16. $x + z - 32k^4 - 32k^6 = 192\,[[\mathrm{Tet}_{k^2}]$
17. $z - 96\,[\mathrm{SP}_{k^2}] - 16k^4 \equiv 0 \pmod{48}$.
18. $y + z - 16\,[\mathrm{SP}_{k^2}] - 80k \equiv 0 \pmod{112}$.
19. $x + y - 16\,[\mathrm{SP}_{k^2}] - 48k^4 \equiv 0 \pmod{80}$.
20. $x + y - 16\,[\mathrm{SP}_{k^2}] - 80k^4$ = 48 times a perfect square.

PATTERN II

Choosing

$Y = \alpha^3$ (7)

$X - Y = 4\alpha$ (8)

in (4), and solving we get

$X = \alpha^3 + 4\alpha$

$Y = \alpha^3$

In view of (3) the integral values p and q are given by

$$p = \alpha^3 + 2\alpha,\ \ q = 2\alpha$$

where in α can take any positive integer ($\alpha > 0$)

Thus the corresponding sides of the Pythagorean triangle obtained from (1) are given by

$x = x(\alpha) = 4\alpha^4 + 8\alpha^2$

$y = y(\alpha) = \alpha^6 + 4\alpha^4$

$z = z(\alpha) = \alpha^6 + 4\alpha^4 + 8\alpha^2$

Properties

1. $z - y$ = 8 times a perfect square.
2. $3(z - y)$ = a nasty number
3. $y - z + x \equiv 0 \pmod{4}$.
4. $x - 8\ T_{3,\alpha^2} \equiv 0 \pmod{4}$
5. $z - T_{3,\alpha^2} - \alpha^6 \equiv 0 \pmod{4}$
6. $y - 2\alpha^2\ T_{3,\alpha^2} \equiv 0 \pmod{3}$.
7. $z - 6\,[\mathrm{Tet}_{\alpha^2}] - \alpha^4 \equiv 0 \pmod{3}$.
8. $x + y - 6\,[\mathrm{Tet}_{\alpha^2}] - 5\alpha^4 \equiv$ 6 times a perfect square.
9. $x + y - 6\,[\mathrm{Tet}_{\alpha^2}] - 6\alpha^2 \equiv 0 \pmod{5}$
10. $x + y - 6\,[\mathrm{Tet}_{\alpha^2}] - 5\alpha^4 \equiv 0 \pmod{8}$
11. $z - y \equiv 0 \pmod{8}$
12. $y + z - 6\,[\mathrm{SP}_{\alpha^2}] - 5\alpha^4 \equiv 0 \pmod{7}$
13. $2(x+y) - 6\,[\mathrm{SP}_{\alpha^2}] - 13\alpha^4 \equiv 0 \pmod{15}$
14. $2z - 6\,[\mathrm{SP}_{\alpha^2}] - \alpha^4 \equiv 0 \pmod{5}$

PATTERN III

Choosing

$Y = 2\alpha$ (9)

$X - Y = 2\alpha^3$ (10)

in (4) and solving we get

$X = 2\alpha + 2\alpha^3$

$Y = 2\alpha$

In view of (3), the integral values of p and q are given by

$p = 2\alpha + \alpha^3,\ \ q = \alpha^3$

where α can take any positive integer ($\alpha > 0$).

Thus, the corresponding sides of the Pythagorean triangle are given by

$x = x(\alpha) = 4\alpha^4 + 2\alpha^6$

$y = y(\alpha) = 4\alpha^2 + 4\alpha^4$

$z = z(\alpha) = 4\alpha^2 + 4\alpha^4 + 2\alpha^6$

Properties

(1) $z - y \equiv 0 \pmod{2}$
(2) $y - z + x \equiv 0 \pmod{4}$
(3) $z - x$ = 4 times a perfect square
(4) $6(z - x)$ is a nasty number.
(5) $x - 4\,(T_{3,\alpha^2}) \equiv 0 \pmod{2}$
(6) $z - x \equiv 0 \pmod{4}$
(7) $y - 8\,(T_{3,\alpha^2}) = 0$
(8) $x - 4\,(T_{3,\alpha^2}) \equiv 0 \pmod{2}$
(9) $y - 4\alpha^2 \equiv 0 \pmod{4}$
(10) $z - 12\,[\mathrm{Tet}_{\alpha^2}] \equiv 0 \pmod{2}$
(11) $x + y - 12\,[\mathrm{Tet}_{\alpha^2}] \equiv 0 \pmod{2}$
(12) $y + z - 12\,[\mathrm{Tet}_{\alpha^2}] - 2\alpha^4 \equiv$ 4 times a perfect square
(13) $y + z - 12\,[\mathrm{Tet}_{\alpha^2}] - 4\alpha^2 \equiv 0 \pmod{2}$
(14) $y + z - 2\alpha^4 - 12\,[\mathrm{Tet}_{\alpha^2}] \equiv 0 \pmod{4}$
(15) $z - 6\,[\mathrm{SP}_{\alpha^2}] + \alpha^4 \equiv 0 \pmod{3}$
(16) $x + y - 6\,[\mathrm{SP}_{\alpha^2}] - 5\alpha^4 \equiv 0 \pmod{7}$

PATTERN IV

Choosing

$Y = 2\alpha^3$ (11)

$X - Y = 2\alpha$ (12)

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in (4) and solving we get

$$X = 2\alpha + 2\alpha^3$$
$$Y = 2\alpha^3$$

In view of (3), the integral values of p and q are given by

$$p = 2\alpha^3 + \alpha, \quad q = \alpha$$

where α can take any positive integer.

Thus, the corresponding sides of the Pythagorean triangle obtained from (1) are given by

$$x = x(\alpha) = 2\alpha^2 + 4\alpha^4$$
$$y = y(\alpha) = 4\alpha^4 + 4\alpha^6$$
$$z = z(\alpha) = 2\alpha^2 + 4\alpha^4 + 4\alpha^6$$

Properties

(1) $z - y = 2$ times a perfect square.

(2) $3(z - y)$ is a nasty number.

(3) $y - z + x \equiv 0 \pmod 4$

(4) $z - 2\,[Tet_{\alpha^2}] - 2\alpha^6 + 4\,T_{3,\alpha^2} = 0$

(5) $y - 8\alpha^2\,T_{3,\alpha^2} = 0$

(6) $x - 4\alpha^2\,T_{3,\alpha^2} \equiv 0 \pmod 2$

(7) $y - x \equiv 0 \pmod 2$

(8) $(x + z - 12\,[SP_{\alpha^2}]$

$-2\alpha^4 \equiv 0 \pmod 4$

(9) $(x + z) - 12\,[SP_{\alpha^2}] - 4\,P\rho_n = 0$

(10) $(x + z) - 2\,[Tet_{\alpha^2}] - 4\,[P\rho_n] - 2\alpha^6 \equiv 0$ (mod 2)

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